How to Generate Chaos at Home

Most people try to avoid chaos. After all, who wants to be stuck in an airport on a snowy afternoon before Christmas? But I find chaos relaxing, particularly when I can observe and control it. Chaos is part of the beauty of nature. It can be observed in the flow of a river, the swing of a pendulum or the dynamics of a cloud [see "Quantum Chaos," by Martin C. Gutzwiller, page 78].

One of the best ways to experiment with chaos is to build an electronic circuit such as the one depicted in the diagram below. The circuit serves as a paradigm for chaotic systems. When the circuit is subjected to certain voltages, it produces a signal that is chaotic.

In 1981 Paul S. Linsay of the Massachusetts Institute of Technology was the first to study rigorously the circuit's behavior. Since then, many physicists have attempted to explain how the circuit generates chaos.

When I learned about Linsay's experiments, I was struck by the fact that a simple circuit could produce such complicated and interesting behavior. I had seen computer simulations of chaotic systems, but here was a chance to study the chaotic dynamics of nature firsthand. As an amateur scientist, I decided I wanted to see chaos for myself. And with a little bit of effort, I even found a way to listen to it.

To construct the circuit, all you need is some basic electronic components—a resistor, an inductor and a diode. I recommend that you start with a resistor of 200 ohms and an inductor of 100 millionths of a henry, the unit of inductance. I have found that many different diodes will work, including 1N4001, 1N4004, 1N4005 and 1N4007.

You can buy a handful of resistors, inductors and diodes for less than $20. I purchased the components from a mail-order company. You can also try a hobby store that carries electronics.

The circuit can be assembled on a breadboard, which is a plastic block with rows of holes to accommodate components. The board costs about $10. To supply the input signal, you will need a function generator, which can be bought for around $200. To measure and view the output of the circuit, you are best off using an oscilloscope. If you do not own an oscilloscope and a function generator, you can probably arrange to use them at a department of physics or engineering at a local college.

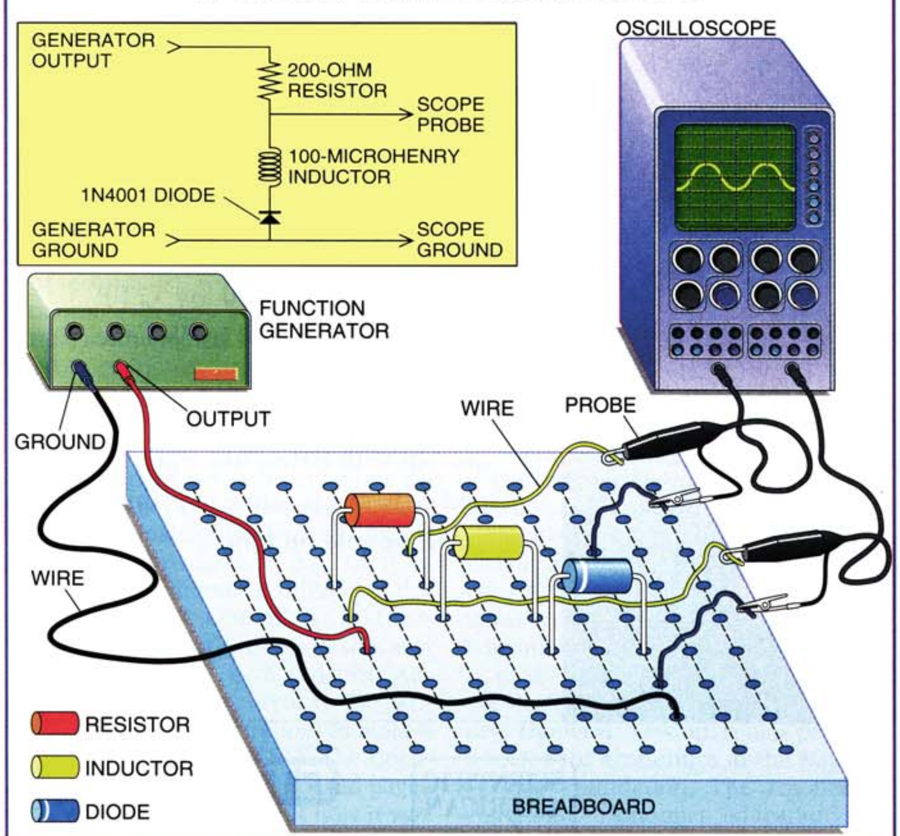
I wanted to have a scope of my own so that I could perform experiments at home, but I could not afford to pay the $1,000 for a new one. Hoping to buy a used scope, I posted a notice on a bulletin board at Argonne National Laboratory. A week later I was contacted by someone who repairs fire alarms and happened to have an oscilloscope for sale. He invited me to his workshop to take a look. The room was filled with electronic parts: dismantled computers, old antennas, disassembled radios, stacks of power supplies. I knew I was in the right place. He sold me a 15-year-old oscilloscope for $100.

Once you have obtained all the equipment, it will not take long to assemble the circuit. The components are connected in series: first the function generator, then the resistor, then the inductor and finally the diode.

You might be aware that the orientation of a diode is an important consideration when building a circuit. A diode has two terminals, known as the cathode and the anode. The cathode terminal is almost always marked by a band on the diode. Ideally, a diode allows current to flow only from cathode to the anode and not the other way.

When you build the chaos circuit for the first time, I recommend that you connect the cathode terminal to the inductor and the anode to ground. But if you do insert the diode the other way, you will find that it makes very little difference.

A Circuit That Produces Chaos



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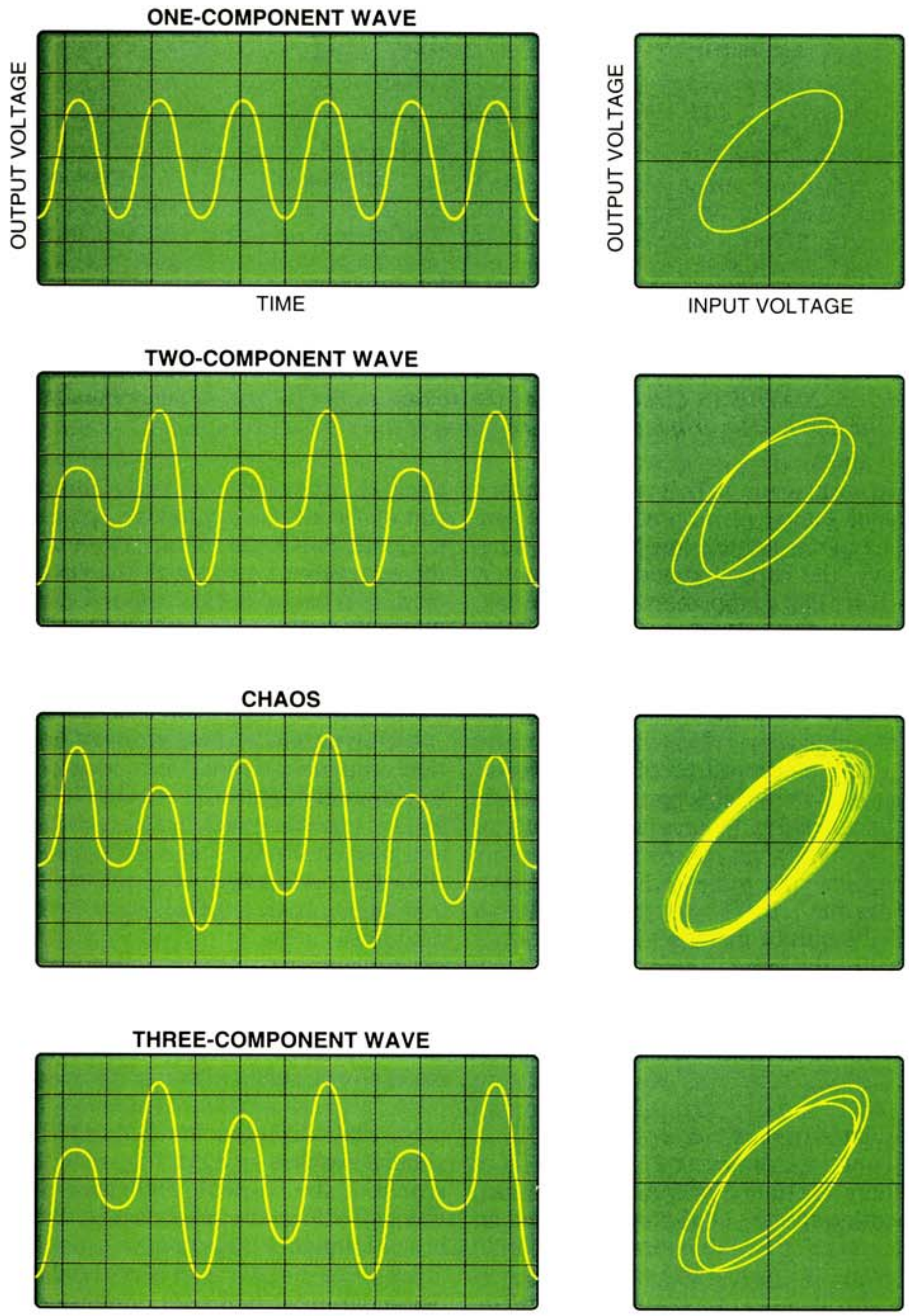
Response of Chaos Circuit



To monitor the output of the circuit, clip an oscilloscope probe between the resistor and the inductor. If your scope has a second probe, use it to measure the input. Check all connections and make sure that all components are firmly seated in the breadboard.

You are now ready to explore chaos. Set the function generator so that it produces sine waves whose frequency is about two million cycles per second (hertz) and whose amplitude varies from 0.1 to -0.1 volt. The output signal should have a lower amplitude than the input but the same frequency. Slowly increase the amplitude of the input signal. At a certain amplitude between one and two volts, the circuit will suddenly produce an output signal with peaks of two different heights. The signal actually consists of two components, each having a different frequency. The point at which new components are introduced is known as a bifurcation.

As you continue to increase the amplitude by small discrete amounts, the signal bifurcates again and again. If you record the amplitude at which each bifurcation occurs, you will notice that change in amplitude between bifurcations decreases geometrically. At a certain amplitude, the system will have bifurcated an infinite number of times, thereby achieving chaos. The signal is not random but a complicated mixture of components.

By increasing the amplitude beyond the onset of chaos, you should be able to produce an output signal with three or even five frequency components. This effect is typical of chaotic systems.

If your circuit is not generating chaos, tune the function generator to a different frequency and try again. If you still don't see it and if you are sure everything else is working properly, you should try a different kind of diode. I have learned that the diodes that work best are ones that have a high capacitance. You can find out the capacitance of the diode by calling the manufacturer or by obtaining a copy of the diode's "data sheet."

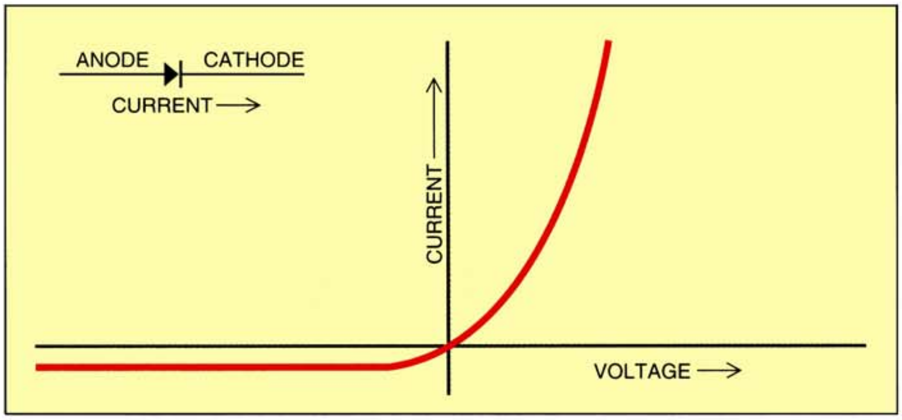
You might also find it difficult to experiment at frequencies of a few million hertz because of the limitations of your function generator or oscilloscope. By changing the basic components, you can work at lower frequencies. If you use a 1N2858 diode, a resistor of 25 ohms and an inductor of 0.1 henry, you can drive the circuit into chaos around 75,000 hertz. You may be forced to order the 0.1-henry inductor from a catalogue since they are rarely used by hobbyists.

The source of the chaotic behavior in the circuit is the diode. Ideally, a diode would conduct current in only one direction. Current will flow through the diode only if the voltage at the cathode is lower than that at the anode. In the circuit the anode is connected to ground (zero volts); therefore, if the cathode has a negative voltage with respect to ground, the diode will conduct.

Real diodes differ from the ideal. If the voltage at the cathode is positive, the diode acts somewhat like a capacitor, that is, the voltage across the capacitor is proportional to the rate of change of the current. It will continue to act in this manner until the voltage at the cathode reaches about -0.5 volt. Then it will freely conduct current, but it will still resist the flow somewhat, maintaining the voltage at about -0.5 volt. If the voltage at the cathode then drops below zero volts, it does not shut off immediately. For a short time, the diode allows the current to flow and then acts like a capacitor again.

Circuits that require diodes are usually designed so that the nonideal properties of the diode are suppressed. But our experimental circuit brings out the full character of the diode.

First consider what happens if the input of the circuit is a sine wave whose amplitude varies from 0.1 to -0.1 volt. Under these conditions, the voltage at the cathode does not go below -0.5 volt, and therefore the diode behaves



like a capacitor. In this case, the behavior of the circuit is predictable. When the input is set to a low-frequency sine wave, the capacitor and the inductor will act like a large resistor, preventing current from flowing through them. The amplitude of the input wave will thus equal that of the output wave.

As the frequency of the input is increased to some critical value, the inductor and diode will provide little resistance, thereby allowing current to flow through them to ground. The output voltage will then approach zero. As the frequency is increased or decreased from the critical value, the amplitude of the output increases until it equals that of the input.

The critical frequency depends on the capacitance of the diode and the inductance of the circuit. It equals

$$\frac{1}{2\pi \sqrt{(\text{inductance} \times \text{capacitance})}}$$

where the inductance is measured in henries and the capacitance is given in farads. If the inductance is 100 millionths of a henry and the capacitance is 50 trillionths of a farad, then the critical frequency equals about 2.3 million hertz.

Now if the amplitude varies from +2 to −2 volts, the diode may be behaving in one of two ways. It may allow current through. Or it may behave like a capacitor. Which behavior it chooses now depends on the voltage at its cathode and how long that voltage has been applied. In turn, the applied voltage at the cathode is related to how the inductor reacts to the input voltage. Then again, how the inductor reacts also depends on whether the diode is charging up like a capacitor or is holding at −0.5 volt.

In simple terms, the inductor is receiving one set of instructions from the input signal and another set from the diode. If the sequence and timing of the instructions are just right, the circuit may continue to produce a periodic signal. But if the natural rhythm of the instructions is broken, the circuit produces chaos. During the past decade, Roger W. Rollins and Earle R. Hunt of Ohio University have been working on a computer simulation that describes the behavior of the circuit exactly.

After you get a good grasp of how the components work, I encourage you to experiment with the circuit. For instance, you can insert different diodes and observe how the chaotic behavior changes. Or you might try changing the frequency, shape or DC offset of the input signal.

You might also find it interesting to visualize the output signal in a different way. You can plot the input signal versus the output by using the oscilloscope in the *x-y* mode. (One oscilloscope probe should monitor the input; the other should record the output.) You should see one or more loops on the screen of the scope. In this mode the number of loops increases as the number of frequency components rises.

For those readers who like dramatic demonstrations and who do not have an oscilloscope at hand, I recommend a different sort of experiment. It is possible to listen to chaos by hooking the circuit up to your stereo system. Before you do so, you should think about whether the input to your receiver can handle the output of the circuit. You don't want to blast too much current or voltage into the receiver.

You should first confirm that your function generator is producing the voltages that you desire. If the input voltage to the circuit is five volts and the resistor is 200 ohms, the maximum current should be 25 milliamps. (The current equals voltage divided by resistance.) By consulting the owner's manual to your stereo, you should be able to discover whether the input channels can handle five volts and 25 milliamps. If not, you can increase the resistance of the circuit.

To connect the circuit to the stereo, use a patch cord. If you cut off an end of the cord and strip it, you will see a wire and either a metallic shielding or a second wire. Connect the first wire to the output of the circuit. Then attach a wire from the shielding to the ground of the circuit, or connect the second wire to ground. Make sure the stereo is turned off and then insert the plug into an input channel of the stereo.

First, set the function generator to sine waves of one volt at about 1,000 hertz. Turn the volume to a low setting and then switch on the stereo. You should hear a tone about two octaves above middle C. As you increase the frequency, the tone should rise in pitch. But when you reach about 20,000 hertz, the tone will be out of your hearing range. Next turn the frequency up to two million hertz. You should not hear a tone.

To drive the circuit into chaos, increase the amplitude of the input slowly. Do not turn up the voltage beyond five volts unless your stereo can handle more than 25 milliamps. As the amplitude increases, the circuit will produce a signal with two frequency components, then four, then eight and so on. You should still hear no sound. When the amplitude increases beyond the onset of chaos, however, the stereo should hiss loudly. When the circuit behaves chaotically, it generates a wide range of frequency components, including some that you can hear.

You can now explore chaos with your ears. In general, the greater the amplitude of the input signal, the more frequency components you will hear. At certain amplitudes, however, the circuit will generate a signal with only three or five frequency components, and the noise will stop.

I do not think the chaotic circuit has much of a future as a musical instrument. But who knows? Composers have written symphonies using electronic synthesizers. Why not a concerto for chaotic circuit in C major?

FURTHER READING

PERIOD DOUBLING AND CHAOTIC BEHAVIOR IN A DRIVEN ANHARMONIC OSCILLATOR. Paul A. Linsay in *Physical Review Letters*, Vol. 47, No. 19, pages 1349–1352; November 9, 1981.

CHAOS: MAKING A NEW SCIENCE. James Gleick. Viking Penguin, 1987.

THE ART OF ELECTRONICS. Paul Horowitz and Winfield Hill. Cambridge University Press, 1989.